Dilations Investigation—Student Activity

Title: Dilation Investigation

Subject: Coordinate Transformations in Geometry

Objective: Given grid paper, a centimeter ruler, a protractor, and a sheet of patty paper the students will generate and apply the relationship between the scale factor of dilation and the coordinates of the image, angle measures, perimeter, and area by completing data tables and by redrawing figures on graph paper.

TEKS Focus: 8.3ABC, 8.10D

(3) Proportionality. The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to:

   (A) generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation;

   (B) compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane; and

   (C) use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation.

(10) Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

   (D) model the effect on linear and area measurements of dilated two-dimensional shapes.

Lesson Overview: In part one, students will use patty paper to create a dilation with a scale factor of two on plain paper. Students will then transfer the dilation to the coordinate grid and create additional dilations of the same figure with scale factors of three and one-half.

In part two, students will make coordinate dilations of several figures and will look at the effect of the scale change on the coordinates, side length, perimeter, and area of each image.

Extension: Students can graph the coordinates of the four triangles in part one using a graphing calculator and determine the equations for the rays used in the dilation.

Grouping: Students can work individually or in groups.

Materials: Each student will need a copy of the lesson, a separate sheet of one centimeter grid paper, a centimeter ruler, a protractor, and a sheet of patty paper.

Procedure: Students can the use same piece of patty paper for steps one through eight to transfer the dilations to the grid paper. Students should be able to work through the document on their own with teacher guidance.

Homework: Students can complete part two of the assignment as homework. Most students should be able to complete both parts in a ninety minute class period.

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Name: *Answer Key* _____   Geometry

Dilations – Part I

1. On the picture below, draw rays from O through each vertex of $\triangle ABC$.

2. Use patty paper to measure the length of $\overline{OA}$. Place A' on OA so that $O'A' = 2 \bullet OA$

3. Use the same process to find B' and C'.

4. Draw $\triangle A'B'C'$.

5. Trace $\triangle ABC$ on a piece of patty paper. Compare the angles of $\triangle ABC$ with the angles of $\triangle A'B'C'$. What do you notice about the angle measures?

The angle measures are the same.

6. Use a ruler to find the following lengths in centimeters.

   a. $AB = $ *3 cm* _____   d. $A'B' = $ *6 cm* _____

   b. $BC = $ *4 cm* _____   e. $B'C' = $ *8 cm* _____

   c. $AC = $ *5 cm* _____   f. $A'C' = $ *10 cm* _____

7. What do you notice about the lengths of the sides of the two triangles?

The lengths of $\triangle A'B'C'$ are twice as long as the lengths of $\triangle ABC$

8. The **Scale Factor** for this dilation is *2 or 2:1* .
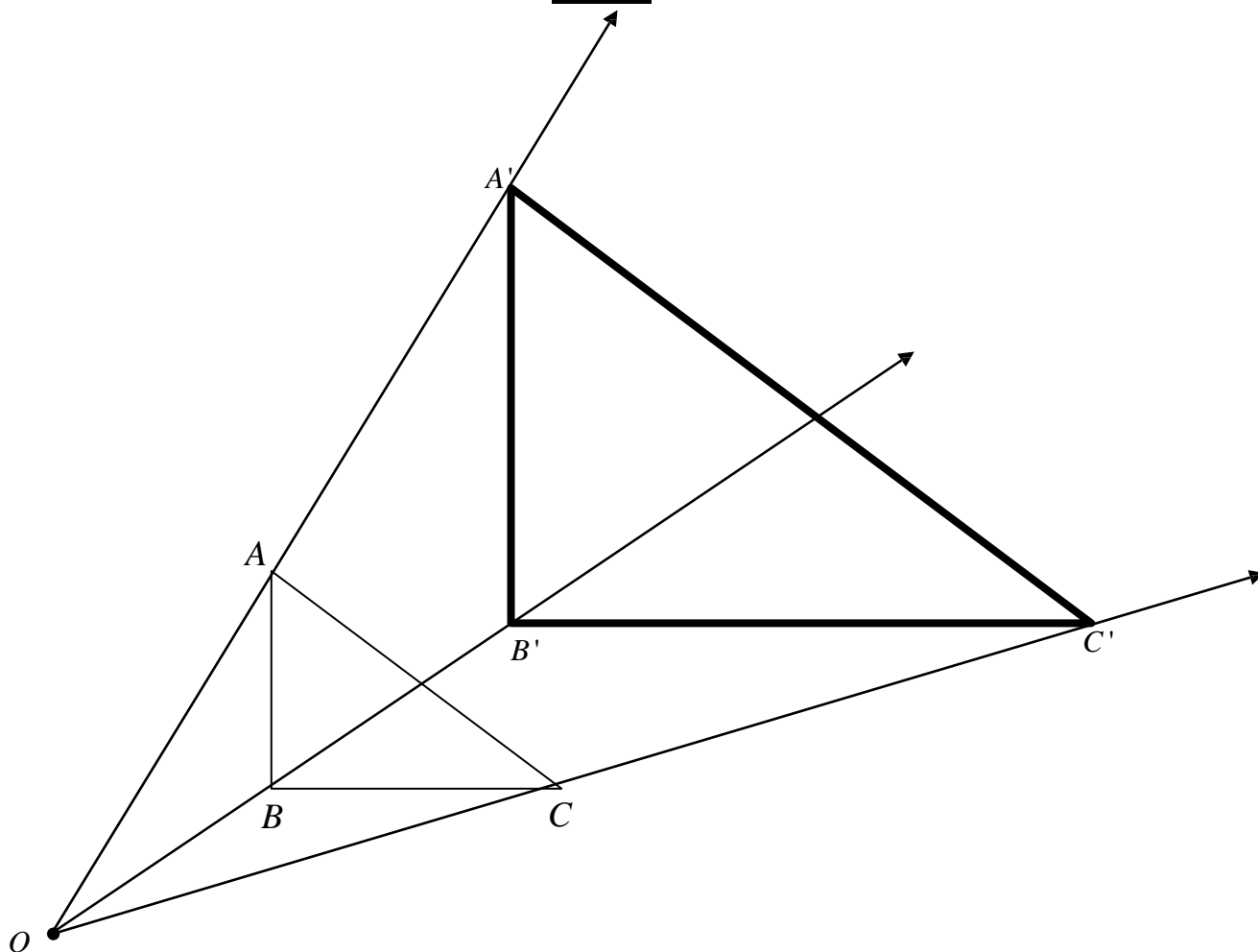
Dilations Investigation—Student Activity

9. Use your patty paper to transfer $\triangle ABC$ and $\triangle A'B'C'$ to the grid paper with point O at the origin. Write the ordered pairs next to each vertex on the grid paper. What do you notice about these ordered pairs?

The coordinates of $\triangle A'B'C'$ are twice as big as the coordinates of $\triangle ABC$

10. Dilate $\triangle ABC$ using a scale factor of 3 on your grid paper. Label the new triangle $\triangle A'B'C'$. Write the ordered pairs next to each vertex.

11. Dilate $\triangle ABC$ using a scale factor of $\frac{1}{2}$ on your grid paper. Label the triangle $\triangle A'B'C'$. Write the ordered pairs next to each vertex.

Table 1 – Ordered Pairs

Original Triangle		Dilation (Scale Factor of 2)		Dilation (Scale Factor of 3)		Dilation (Scale Factor of $\frac{1}{2}$)	
A	*(3, 5)*	A'	*(6, 10)*	A''	*(9, 15)*	A'''	*(1.5, 2.5)*
B	*(3, 2)*	B'	*(6, 4)*	B''	*(9, 6)*	B'''	*(1.5, 1)*
C	*(7, 2)*	C'	*(14, 4)*	C''	*(21, 6)*	C'''	*(3.5, 1)*

12. What observations/conjectures can you make about dilations based on your work so far?

Possible answers:
When the scale factor is two, you multiply the coordinates by 2. When the scale factor is three, you multiply the coordinates by three. When the scale factor is $\frac{1}{2}$, you multiply the coordinates by $\frac{1}{2}$.

13. Complete this conjecture based on Table 1: The image of (a, b) after a dilation with center O and scale factor k has coordinates $\underline{(ka ,\ kb)}$.

Name: _____   Geometry

Dilations – Part II

1. Record your observations about the characteristics of the triangles $\triangle ABC$ and $\triangle A'B'C'$ in the table below.

Characteristics	Original $\triangle ABC$	$\triangle A'B'C'$ (Scale Factor of 2)	Observations
Coordinates	A(3, 5) B(3, 2) C(7, 2)	A(6, 10) B(6, 4) C(14, 4)	Doubled
Angle Measures	53° - 90° - 37°	53° - 90° - 37°	Same measure
Length of Sides	3 cm–4cm–5 cm	6 cm–8 cm–10 cm	Doubled
Perimeter	12 cm	24 cm	Doubled
Area	6 cm²	24 cm²	Quadrupled

2. Graph Quadrilateral $JKLM$ on the back of your grid paper. Dilate quadrilateral $JKLM$ with a scale factor of 3 on your grid paper. Label the image $J'K'L'M'$ and complete the table below.

Characteristics	Original $JKLM$	Quad. $J'K'L'M'$ (Scale Factor of 3)	Observations
Coordinates	J(-2, 1) K(-2, 3), L(3, 3), M(3, 1)	J'(-6, 3) K'(-6, 9), L'(9, 9), M'(9, 3)	Tripled
Angle Measures	90°- 90°- 90°- 90°	90°- 90°- 90°- 90°	Same measure
Length of Sides	2 cm by 5 cm	6 cm by 15 cm	Tripled
Perimeter	14 cm	42 cm	Tripled
Area	10 cm²	90 cm²	Multiplied by 9

3. Graph Quadrilateral $QRST$ on the same grid paper. Dilate quadrilateral $QRST$ with a scale factor of $\frac{1}{2}$ on your grid paper. Label the image $Q'R'S'T'$ and complete the table below.

Characteristics	Original $QRST$	Quad. $Q'R'S'T'$ (Scale Factor of $\frac{1}{2}$)	Observations
Coordinates	Q(-2,-2) R(-2, -4), S(-6, -4), T(-6, 2)	Q'(-1, -1) R'(-1, -2), S'(-3, -2), T'(-3, 1)	One half
Angle Measures	135°- 90°- 90°- 45°	135°- 90°- 90°- 45°	Same measure
Length of Sides	2 cm-4 cm–6 cm-5.7 cm	1 cm-2 cm–3 cm-2.9 cm	One half
Perimeter	17.7 cm	8.8 cm	One half
Area	16 cm²	4 cm²	One fourth

Dilations Investigation—Student Activity

4. A diameter of a circle has endpoints A and B. Graph the circle on your grid paper. Dilate the circle with a scale factor of 2. Label the coordinates of the image A' and B' and complete the table below.

Characteristics	Original circle with diameter $\overline{AB}$	New circle with diameter $\overline{A'B'}$	Observations
Coordinates	$A(4,1)$, $B(4,-3)$	$A'(8,2)$, $B'(8,-6)$	*Doubled*
Length of the diameter	*4 cm*	*8 cm*	*Doubled*
Length of the radius	*2 cm*	*4 cm*	*Doubled*
Circumference	$4\pi \approx 12.56\ cm$	$8\pi \approx 25.12\ cm$	*Doubled*
Area	$4\pi \approx 12.56\ cm^2$	$16\pi \approx 50.24\ cm^2$	*Quadrupled*

5. Based on the information in all four tables, what conjectures can you make about:

a. Scale Factor used in the dilation and angle measures. *Scale factor has no effect on angle measure.*

b. Scale Factor used in the dilation and side lengths/diameter. *Side length is multiplied by the scale factor.*

c. Scale Factor used in the dilation and perimeter/circumference. *Perimeter is multiplied by the scale factor.*

d. Scale Factor used in the dilation and area. *Area is multiplied by the scale factor squared.*

6. Use the graph to below to answer the following dilation questions.
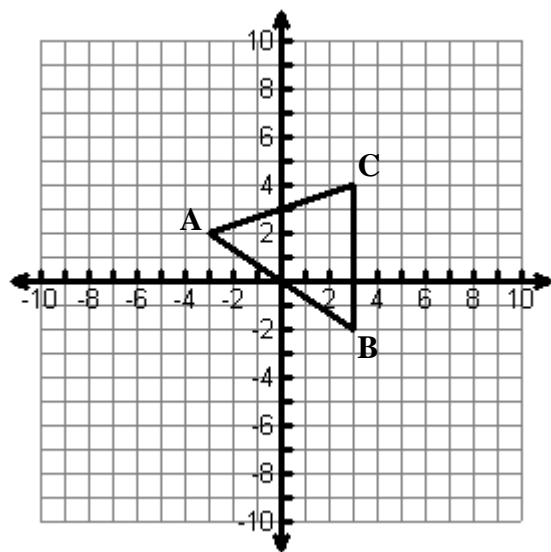
a) Find the coordinates of triangle $A'B'C'$ after a dilation with a scale factor of 3.

$\underline{A'(-9,6)\ B'(9,-6)\ C'(9,12)}$

How does this change angle measures?

$\underline{Angles\ are\ the\ same.}$

b) Find the coordinates of triangle $A'B'C'$ after a dilation with a scale factor of $\dfrac{3}{2}$.

$\underline{A'(-4.5,\ 3)\ B'(4.5,\ -3)\ C'(4.5,\ 6)}$

How does this change the lengths of the sides?

$\underline{Side\ length\ is\ 3/2\ bigger.}$

c) If the coordinates of A' are (-1.5, 1) after a dilation, find the coordinates of B' and C'?

$\underline{B'(1.5,\ -1)\ C'(1.5,\ 2)}$

How does this change the perimeter of the triangle? $\underline{The\ perimeter\ is\ half\ as\ big.}$

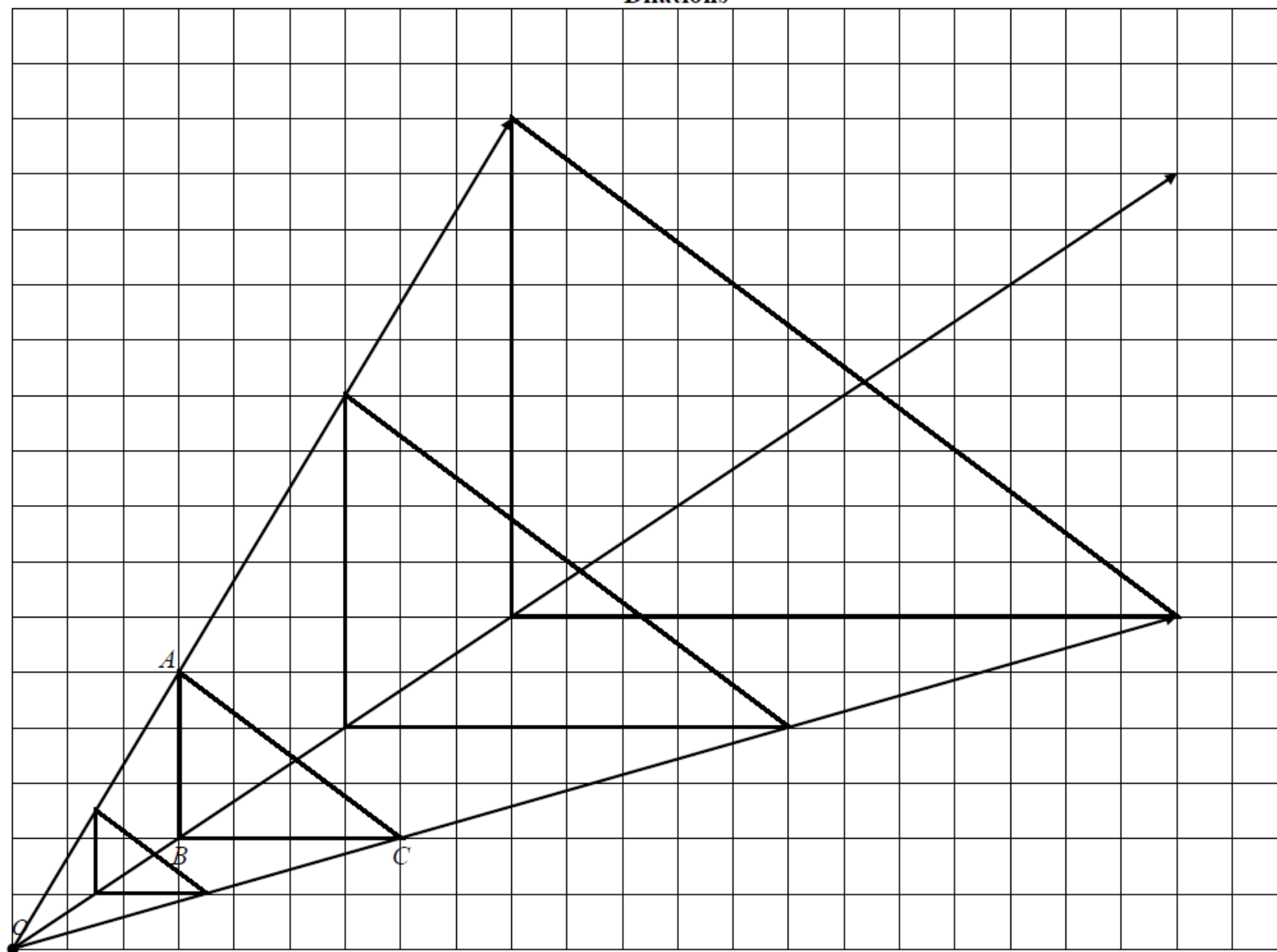d) If the coordinates of B' are (18, -12) after a dilation, find the coordinates of A' and C'?
$A'(-12, 8)\ C'(12,16)$

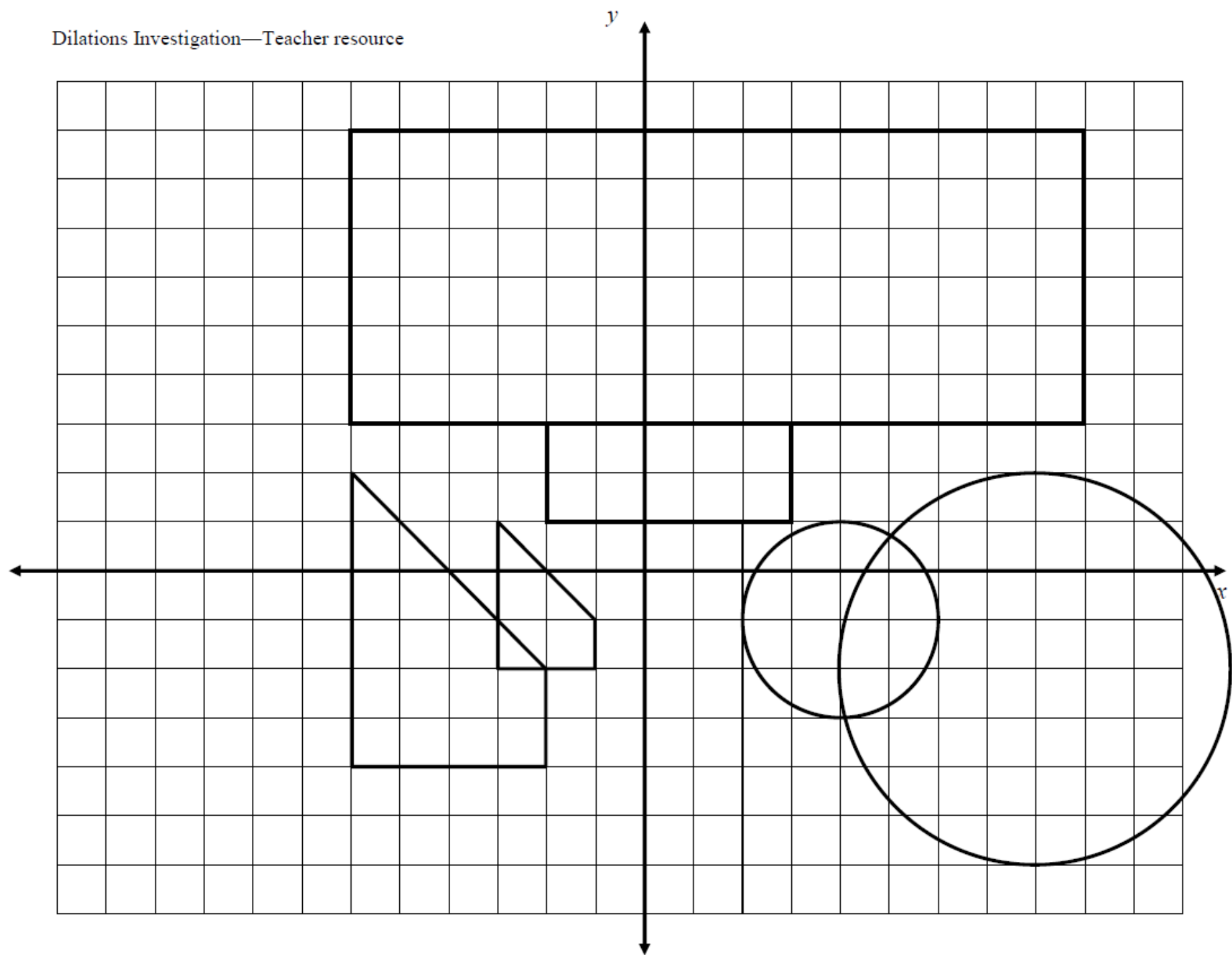How does this change the area of the triangle? $\underline{The\ area\ is\ 6^2\ or\ 36\ times\ bigger.}$

Dilations Investigation—Teacher resource

Name:_____   Geometry

Dilations

Dilations Investigation—Teacher resource

Dilations Investigation—Student Activity

Name: _____   Geometry

Dilations – Part I

1. On the picture below, draw rays from O through each vertex of $\triangle ABC$.

2. Use patty paper to measure the length of $\overline{OA}$. Place A' on OA so that $O'A' = 2 \bullet OA$

3. Use the same process to find B' and C'.

4. Draw $\triangle A'B'C'$.

5. Trace $\triangle ABC$ on a piece of patty paper. Compare the angles of $\triangle ABC$ with the angles of $\triangle A'B'C'$. What do you notice about the angle measures? _____

6. Use a ruler to find the following lengths in centimeters.

     a. $AB = $ _____   d. $A'B' = $_____

     b. $BC = $ _____   e. $B'C' = $_____

     c. $AC = $ _____   f. $A'C' = $_____

7. What do you notice about the lengths of the sides of the two triangles? _____

8. The **Scale Factor** for this dilation is _____.

A

B C
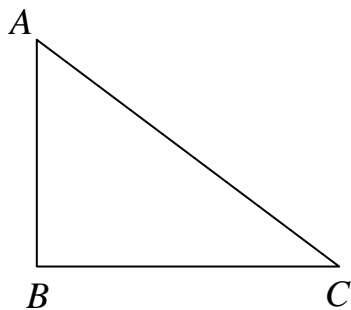
O
•

Dilations Investigation—Student Activity

9. Use your patty paper to transfer $\triangle ABC$ and $\triangle A'B'C'$ to the grid paper with point O at the origin. Write the ordered pairs next to each vertex on the grid paper and in Table 1 below. What do you notice about these ordered pairs?

_____

_____

10. Dilate $\triangle ABC$ using a scale factor of 3 on your grid paper. Label the new triangle $\triangle A'B'C'$. Write the ordered pairs next to each vertex and in Table 1 below.

11. Dilate $\triangle ABC$ using a scale factor of $\frac{1}{2}$ on your grid paper. Label the triangle $\triangle A'B'C'$. Write the ordered pairs next to each vertex and in Table 1 below.

Table 1 – Ordered Pairs

Original Triangle		Dilation (Scale Factor of 2)		Dilation (Scale Factor of 3)		Dilation (Scale Factor of $\frac{1}{2}$)	
A		A'		A''		A'''	
B		B'		B''		B'''	
C		C'		C''		C'''	

12. What observations/conjectures can you make about dilations and ordered pairs based on your work so far?

13. Complete this conjecture based on Table 1: The image of (a, b) after a dilation with center O and scale factor k has coordinates (_____ , _____).

Dilations Investigation—Student Activity

Name: _____   Geometry

Dilations – Part II

1. Record your observations about the characteristics of the triangles $\triangle ABC$ and $\triangle A'B'C'$ from **Dilations – Part I** in the table below.

Characteristics	Original $\triangle ABC$	$\triangle A'B'C'$ (Scale Factor of 2)	Observations
Coordinates	$A(3, 5)$ $B(3, 2)$, $C(7, 2)$		
Angle Measures			
Length of Sides			
Perimeter			
Area			

2. Graph Quadrilateral $JKLM$ on the back of your grid paper (from **Part I**). Dilate quadrilateral $JKLM$ with a scale factor of 3 on your grid paper. Label the image $J'K'L'M'$ and complete the table below.

Characteristics	Original $JKLM$	Quad. $J'K'L'M'$ (Scale Factor of 3)	Observations
Coordinates	$J(-2, 1)$ $K(-2, 3)$, $L(3, 3)$, $M(3, 1)$		
Angle Measures			
Length of Sides			
Perimeter			
Area			

3. Graph Quadrilateral $QRST$ on the same grid paper. Dilate quadrilateral $QRST$ with a scale factor of $\frac{1}{2}$ on your grid paper. Label the image $Q'R'S'T'$ and complete the table below.

Characteristics	Original $QRST$	Quad. $Q'R'S'T'$ (Scale Factor of $\frac{1}{2}$)	Observations
Coordinates	$Q(-2,-2)$ $R(-2, -4)$, $S(-6, -4)$, $T(-6, 2)$		
Angle Measures			
Length of Sides			
Perimeter			
Area			

Dilations Investigation—Student Activity

4. A diameter of a circle has endpoints A and B. Graph the circle on your grid paper. Dilate the circle with a scale factor of 2. Label the coordinates of the image A' and B' and complete the table below.

Characteristics	Original circle with diameter $\overline{AB}$	New circle with diameter $\overline{A'B'}$	Observations
Coordinates	$A(4,1)$, $B(4,-3)$		
Length of the diameter			
Length of the radius			
Circumference			
Area			

5. Based on the information in all four tables, what conjectures can you make about:

    a. Scale Factor used in the dilation and angle measures: _____

    b. Scale Factor used in the dilation and side lengths/diameter: _____

    c. Scale Factor used in the dilation and perimeter/circumference: _____

    d. Scale Factor used in the dilation and area: _____

6. Use the graph to below to answer the following dilation questions.
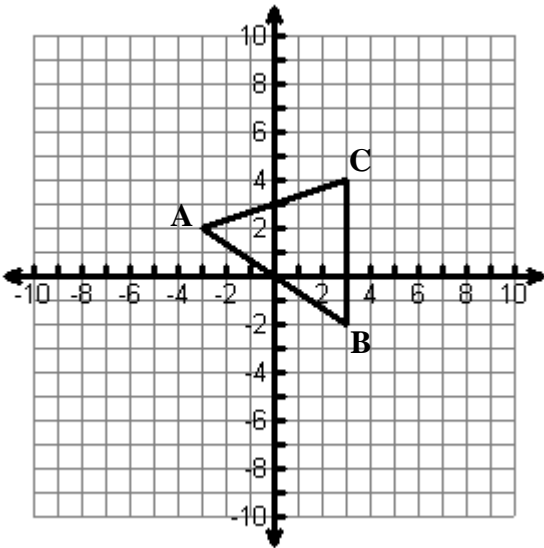
a) Find the coordinates of triangle $A'B'C'$ after a dilation with a scale factor of 3.

_____

How does this change angle measures?

_____

b) Find the coordinates of triangle $A'B'C'$ after a dilation with a scale factor of $\dfrac{3}{2}$.

_____

How does this change the lengths of the sides?

_____

c) If the coordinates of A' are (-1.5, 1) after a dilation, find the coordinates of B' and C'.

_____

How does this change the perimeter of the triangle? _____

d) If the coordinates of B' are (18, -12) after a dilation, find the coordinates of A' and C'. _____
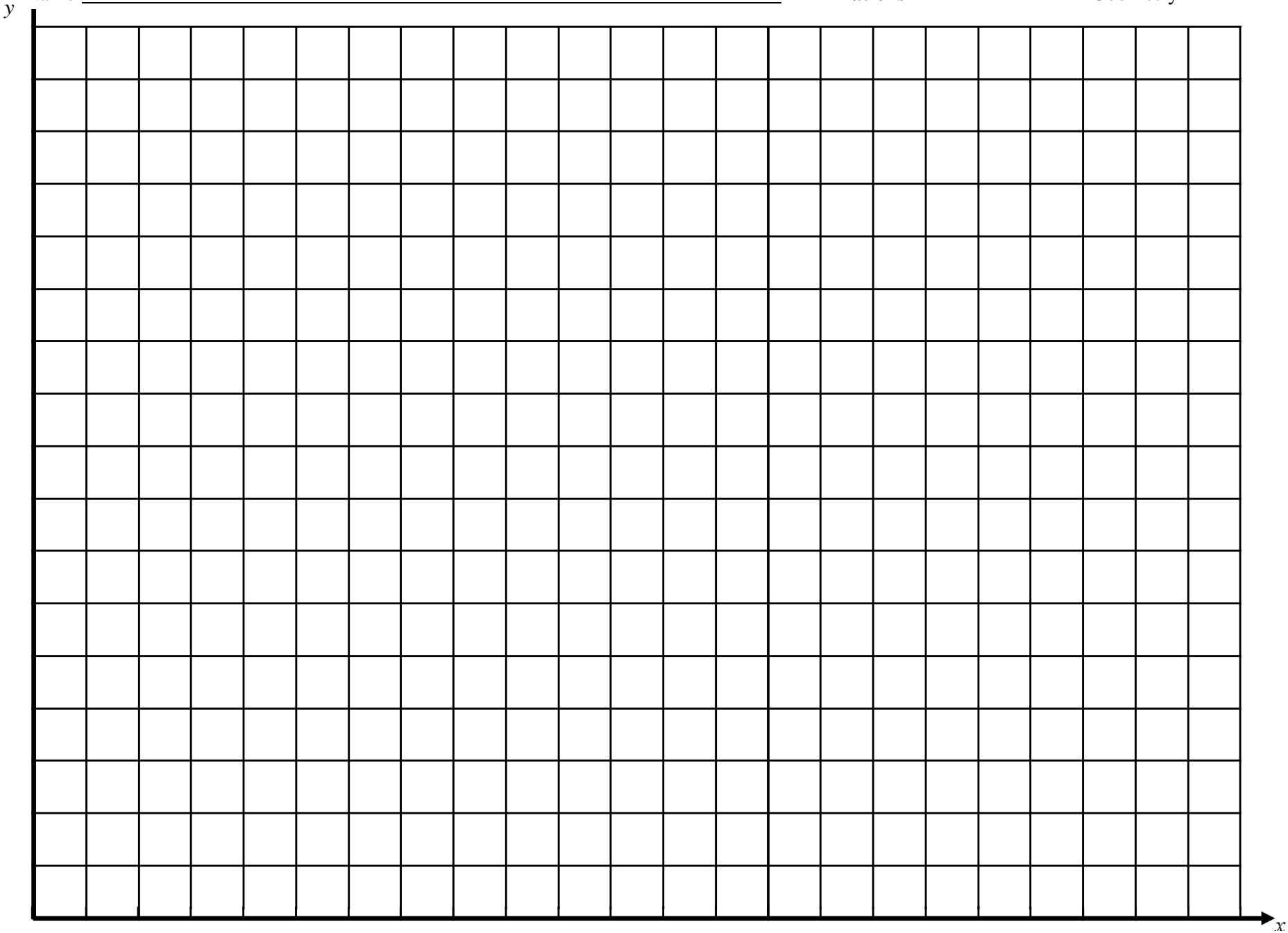
How does this change the area of the triangle? _____
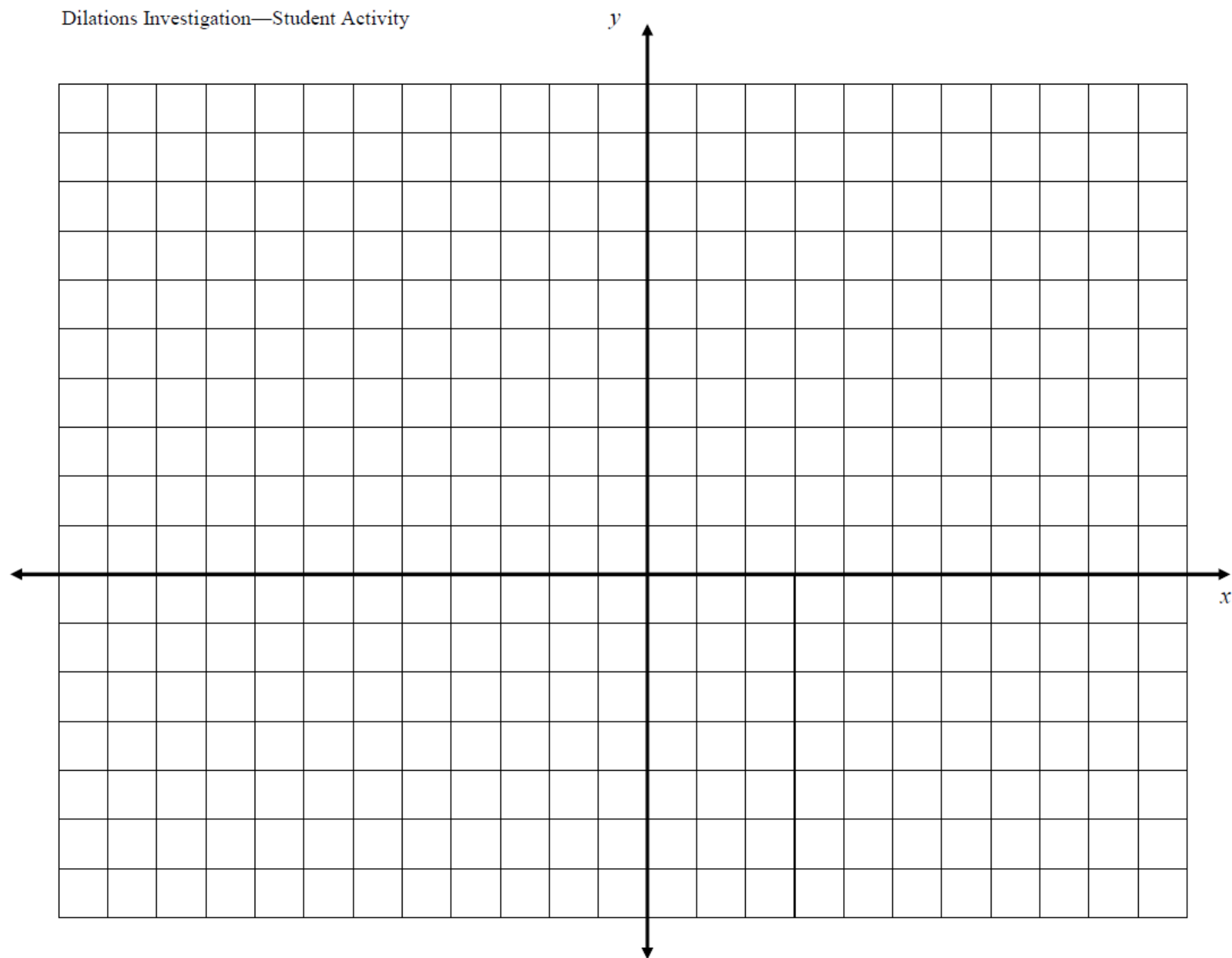
Dilations Investigation-Student Activity
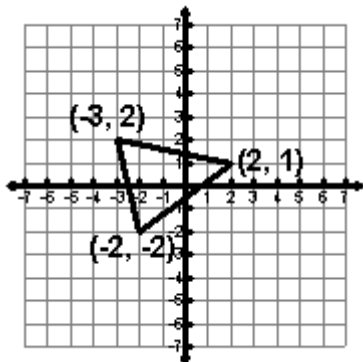
Name _____ **Dilations** Geometry

y

x

Dilations Investigation—Student Activity

Dilations Investigation—Homework
Geometry
Dilation WS

Name: _____

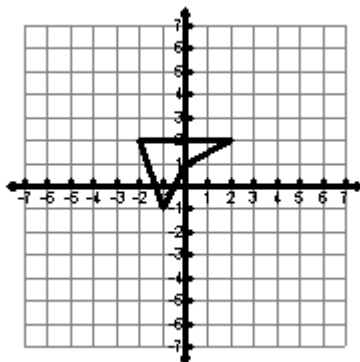Date:_____Per.: _____

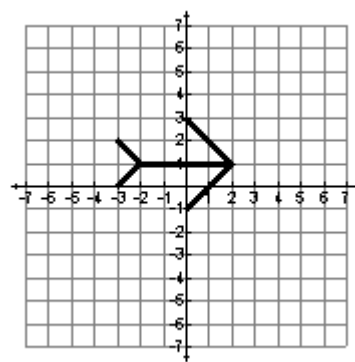## Dilate the following with the given scale factors.
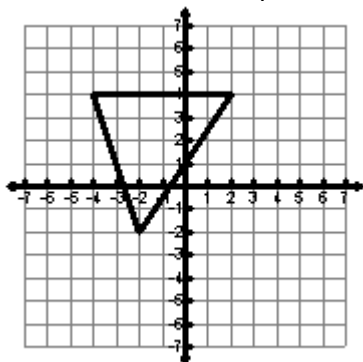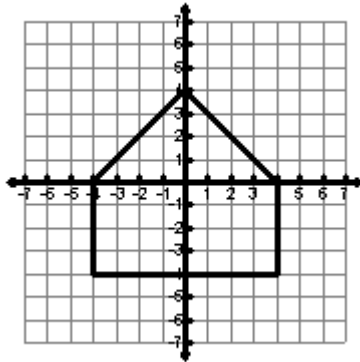
1) scale factor = 2
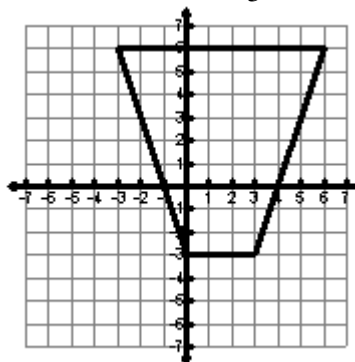

2) scale factor = 3

3) scale factor = 2

4) scale factor = $\dfrac{3}{4}$

5) scale factor = $\dfrac{3}{2}$

6) scale factor = $\dfrac{2}{3}$

Use the problems above to write an algebraic representation to explain the effect of the given scale factors on the coordinates. Ex: $(x, y) \rightarrow (5x, 5y)$

7. Representation for #1: _____

8. Representation for #2: _____

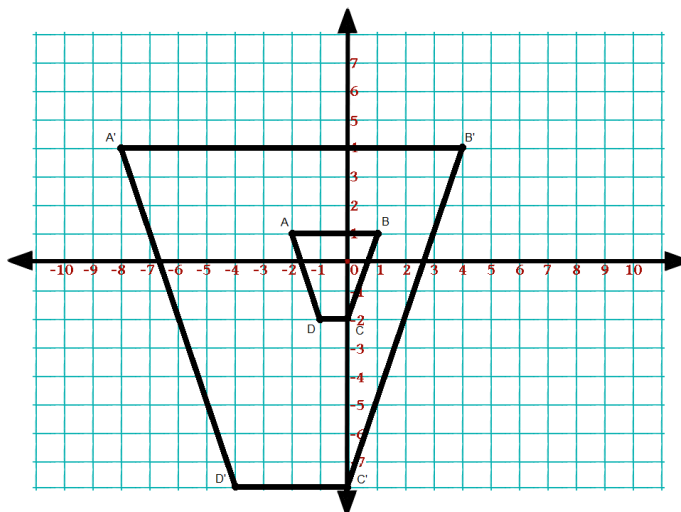9. Representation for #3: _____

10. Representation for #4: _____

11. Representation for #5: _____
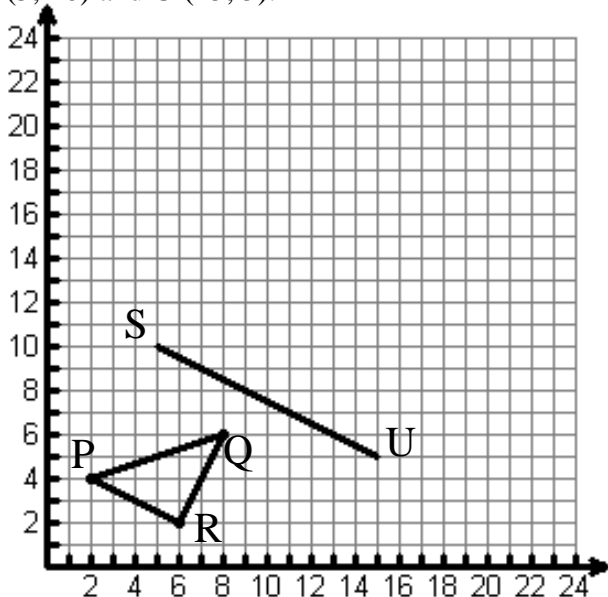
12. Representation for #6: _____

Dilations Investigation—Homework

13) Use the graph below to find the scale factor of the following dilation. _____

a) How does the dilation change the angles measures? _____

b) How does the dilation change the length of the sides? _____

c) How does the dilation change the perimeter? _____

d) How does the dilation change the area? _____

e) How does the dilation change the coordinates?_____

f) Write an algebraic representation to explain the coordinate change. _____

14) The graph shows $\triangle PQR$ with vertices P (2, 4), Q (8, 6), and R (6, 2) and $\overline{SU}$ with endpoints S (5, 10) and U (15, 5).

a) At what coordinates would vertex T be placed to create $\triangle STU$, a triangle that is a dilation of $\triangle PQR$?

F   (12, 9)
G   (16, 12)
H   (20, 15)
J   (24, 18)

b) Write the algebraic representation of this transformation: _____